3314419: Multiobjective Optimization (In English, Fall 2023)

Hall 1.114, Rudower Chausee 25

Thursday: 1 PM - 3 PM, 18th Oct. 2023 - 14th Feb. 2024

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This course is aimed for the target audience of undergraduate students. Several questions in design engineering, automation, and science demand the consideration and optimization of multiple metrics. A very straightforward and relatable example is the requirement to design a car with dual metrics of fuel economy and acceleration ability. On a general note, these problems can be represented as follows.

$$\min_{x \in X} \; F(x) = [f_1(x), \ldots, f_m(x)] \,.$$

In contrast to single objective optimization problems, these problems do not lead to one solution, but a family of solutions. This family of solutions is referred to as the Pareto frontier. Let $\{x_i, F(x_i)\}_{i=1}^{K}$ denote the set of non-dominated points marking the Pareto frontier [ZKT08, ABC$^+$18]. This simply implies that for any two points x_i and x_j, there exists an index k such that $f_k(x_i) \leq f_k(x_j)$ and $f_r(x_i) > f(x_j)$ for atleast one index $r \in \{1, \ldots, K\} - \{k\}$. In simpler terms, this means that improving in one metric comes at the compromise/tradeoff in atleast one of the other metrics. While some non-Pareto solutions have also been discussed in literature, we do not focus on such settings in this course. Our focus will be on understanding the current space of MOO (multiobjective optimization) literature along four different fronts as follows.

- Algorithms - The crux of all the methods predominantly studied so far has been to equivalently solve a series of SOO (single objective optimization) problems in place of our MOO. Here, we note that all standard mathematical programming based methods used for SOO are applicable in this context (post the reformulations). Some standard methods [ASZ08, Ehr05, ER08] are the use of weights, ϵ constraints, and lexicographic methods.
- Metrics - Optimality measures can be multifold in the case of MOO. Some standard metrics include the hypervolume and spread. Here, the attempt is to quantify optimality of points by taking into consideration all the objectives [ABC$^+$18].
- Applications – MOO is very important in several automotive design, energy markets, aircraft design, chemical engineering, portfolio management, and imaging based applications, to name a few. We will cover many such applications.
- Solvers – Some of the MOO type problems are additionally complicated due to the absence of derivatives (due to propriety reasons). Solvers from the standpoint of MOO also mostly attempt to address this issue. Some examples of MOO solvers are ALGLIB, MultiMADS, and USEMO [BDJD20, BLDS21]. We will examine some of these solvers also as a part of this course.

By the end of the course, students would be able to spot real-world situations where MOO arises. They would also be equipped with the basics on the algorithmic and implementation side to help in solving such problems. Several research works (publications) will be distributed during the course of the study. The following texts may be adhered [Ehr05] to as key references to get started with the concepts. The tentative outline of the course is as follows.

- Class 1 – Basics of MOO.
- Class 2 – Following.
  - Some additional background on MOO.
  - Division into smaller (1-2 per group) groups (for reading assignments and question module preparation).
  - Assignment of research works/papers to each of these groups.

- Classes 3-12 – Seminars and questionnaire.
- Classes 13-end – Discussions on prospective and new research directions (both theoretical and algorithmic).

There can also be seminars at regular time slots from external practitioners (Academia/Industry). Some lectures can be held online (more details to be decided later).

Grading: A summary of all presentations and related questions is expected at the end of the semester. The course is graded Satisfactory/Unsatisfactory based on the quality of presentations, understanding of texts, and interactions/participation (summary).

References

[ABC+18] C. Audet, J. Bigeon, D. Cartier, S. Le Digabel, and L. Salomon. Performance indicators in multiobjective optimization. Technical Report G-2018-90, Les cahiers du GERAD, 2018.

[ASZ08] Charles Audet, Gilles Savard, and Walid Zghal. Multiobjective optimization through a series of single-objective formulations. *SIAM Journal on Optimization*, 19(1):188–210, 2008.

[BDJD20] Syrine Belakaria, Aryan Deshwal, Nitthilan Kannappan Jayakodi, and Janardhan Rao Doppa. Uncertainty-aware search framework for multi-objective bayesian optimization. *AAAI Conference on Artificial Intelligence*, 34(06), 2020.

[BLDS21] Jean Bigeon, Sébastien Le Digabel, and Ludovic Salomon. Dmulti-mads: mesh adaptive direct multisearch for bound-constrained blackbox multiobjective optimization. *Computational Optimization and Applications*, 79, 06 2021.

[Ehr05] M. Ehrgott. *Multicriteria Optimization.* Lecture notes in economics and mathematical systems. Springer, 2005.

[ER08] M. Ehrgott and S. Ruzika. Improved -constraint method for multiobjective programming. *Journal of Optimization Theory and Applications*, 138(3):375–396, 2008.

[ZKT08] Eckart Zitzler, Joshua D. Knowles, and Lothar Thiele. Quality assessment of pareto set approximations. In *Multiobjective Optimization*, 2008.