

Topics in geometric analysis - Ricci flow

Intended semester for the course - Summer Semester 2022.

Language - English.

Intended audience - Advanced Masters's students and higher.

Short Description

Right from its introduction by Hamilton in 1982, the Ricci flow has found applications in both geometry and topology. Perhaps the crowning achievement of the Ricci flow is the proof of the Poincaré conjecture or more generally the proof of the Thurston's Geometrization conjecture by Perelman. This course intends to be an introduction to the Ricci flow and to study many of its properties and applications. The target audience is advanced Bachelors and Masters's students and PhD students so only basic knowledge of Riemannian geometry and analysis (especially PDEs) will be very beneficial. A detailed (preliminary) discussion of topics is outlined below. If the response will be good, then there could also be a "Part 2" of the course which probably will cover those results of Perelman which won't be covered in the first part.

Topics to be covered (the topics marked with * might be covered if there is interest among the participants.)

- (1) Basics of Riemannian geometry and Ricci calculus with emphasis on calculations in local coordinates.
- (2) Basics on Partial Differential Equations with a focus on parabolic PDEs; existence of solutions to such PDEs.
- (3) Introduction to the Ricci flow.
- (4) Short time existence using the DeTurck's trick.
- (5) Evolution equations of intrinsic geometric quantities along the flow.
- (6) Uhlenbeck's trick: evolution of the Riemann curvature tensor; Hamilton's theorem on positivity of Riemann curvature being preserved.
- (7) Curvature estimates and long time existence.
- (8) Vector bundle maximum principles; curvature pinching estimates and Hamilton–Ivey pinching estimate.
- (9) Ricci flow in two dimensions. (Hamilton and Ivey's results on all compact Ricci solitons being gradient*)
- (10) Li–Yau Harnack inequality and Hamilton's Harnack estimates for the Ricci flow. (Chow–Chu's approach to Hamilton's Harnack estimates using the space-time approach*)
- (11) Ricci solitons item Ricci flow as a gradient flow: Perelman's \mathcal{F} and \mathcal{W} functionals and their monotonicity.
- (12) Perelman's No Local Collapsing theorem (proof of Hamilton's little loop conjecture.)
- (13) Logarithmic Sobolev inequalities.
- (14) Overall idea of Perelman's proof of the Thurston's Geometrization Conjecture.

More detailed course planning will be announced as the course progresses.

Literature

Even though I'll provide lecture notes, most of it will be derived from a combination of the following materials.

REFERENCES

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- [CK04] Bennett Chow and Dan Knopf, *The Ricci flow: an introduction*, Mathematical Surveys and Monographs, vol. 110, American Mathematical Society, Providence, RI, 2004. MR2061425 ↑
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- [Ham82] Richard S. Hamilton, *Three-manifolds with positive Ricci curvature*, J. Differential Geometry **17** (1982), no. 2, 255–306. MR664497 ↑
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- [Per02] Grisha Perelman, *The entropy formula for the Ricci flow and its geometric applications*, arXiv Mathematics e-prints (November 2002), math/0211159, available at [math/0211159](https://arxiv.org/abs/math/0211159). ↑
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